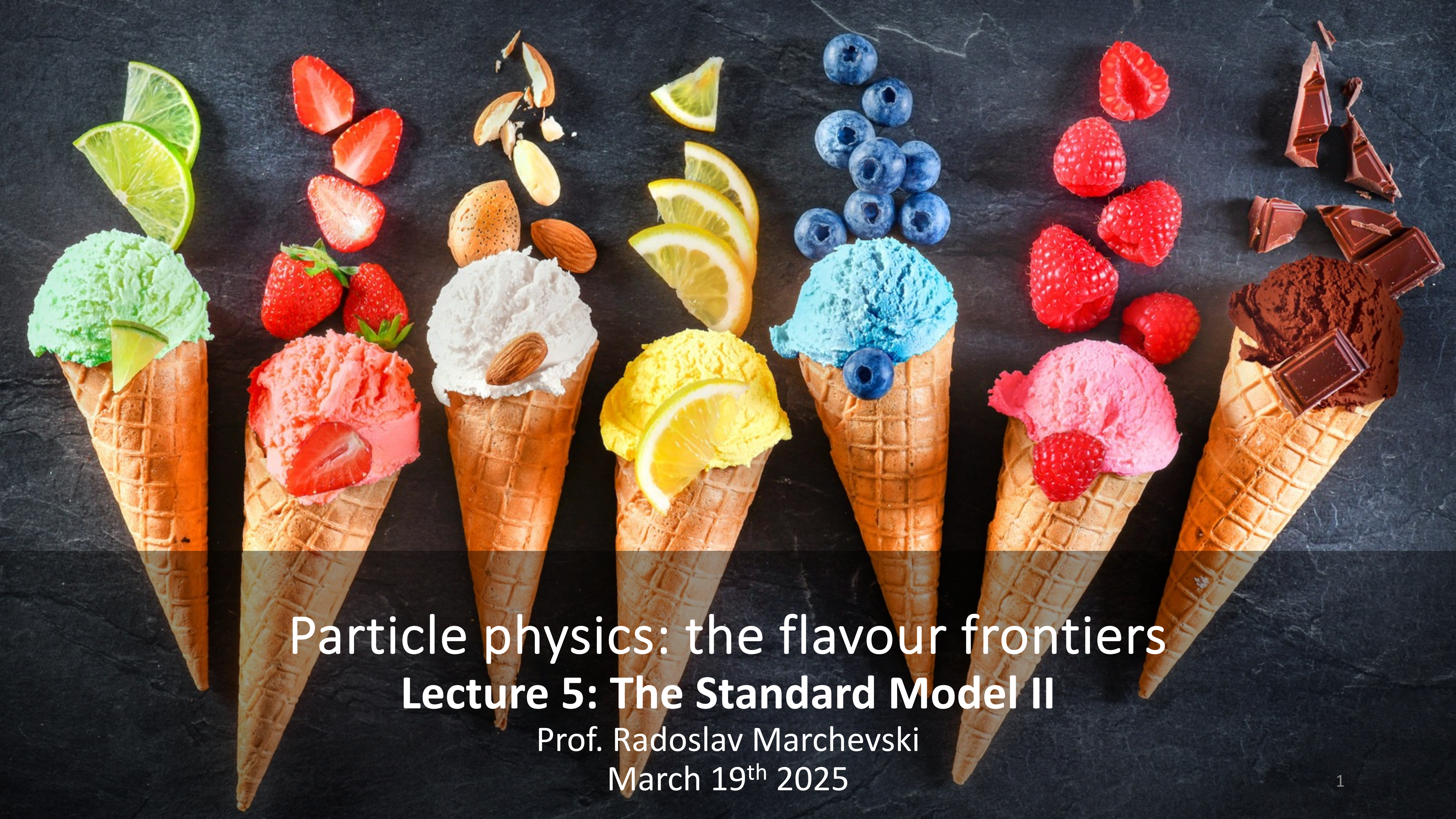

Particle physics: the flavour frontiers

Lecture 5: The Standard Model II

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Short recap and today's learning targets

Last time we discussed

- The Standard Model of particle physics
- We obtained the spectrum of physically observable particles and the mass generation mechanism

Today you will …

- discuss the interactions between the Standard Model fields
- find the global accidental symmetries of the Standard Model
- count the number of parameters necessary to describe the Standard Model

Particles (mass eigenstates)

particle	spin	color	Q	mass $[v]$
$W^{\pm}$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
γ	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e, μ, τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e, ν_μ, ν_τ	1/2	(1)	0	0
u, c, t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d, s, b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

- Mass eigenstates of the Standard Model, their $SU(3)\times U(1)_{EM}$ quantum numbers, and their masses in units of the VEV v
- All masses are proportional to the VEV of the scalar field, v → result of SSB

Electromagnetic (QED) and Strong (QCD) interactions

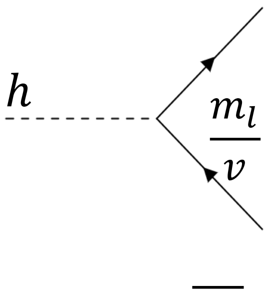
- Photon-mediated electromagnetic interaction are described by Quantum Electro-Dynamics (QED), part of the SM
- We get the following Lagrangian terms for the interaction of the photon field with the SM Dirac fermions

$$\mathcal{L}_{Af\bar{f}} = e\bar{e}_i\gamma^\mu A_\mu e_i - \left(\frac{2}{3}\right) e\bar{u}_i\gamma^\mu A_\mu u_i + \left(\frac{1}{3}\right) e\bar{d}_i\gamma^\mu A_\mu d_i$$

- The gluon-mediated strong interactions are described by QCD, which is part of the SM
- We get the following Lagrangian terms for the interaction of the gluon field with the SM quarks

$$\mathcal{L}_{Gq\bar{q}} = -g_s\bar{q}_i\gamma^\mu G_{a\,\mu}\left(\frac{\lambda_a}{2}\right) q_i$$

The Higgs Boson interactions

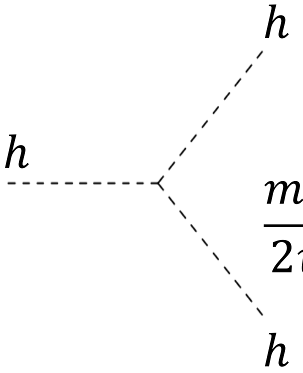


$$\frac{m_l}{v} = \frac{y_l}{\sqrt{2}}$$

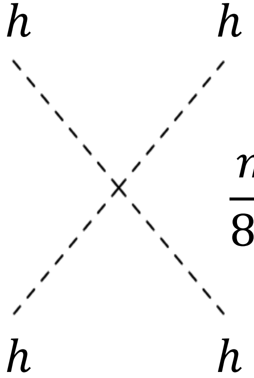
$$\mathcal{L}_{\text{int}}^h = \mathcal{L}_h^h + \mathcal{L}_V^h + \mathcal{L}_f^h$$

$$-\mathcal{L}_f^h = \frac{h}{v}\left(m_e \overline{e_L} e_R + m_\mu \overline{\mu_L} \mu_R + m_\tau \overline{\tau_L} \tau_R + m_u \overline{u_L} u_R + m_c \overline{c_L} c_R + m_t \overline{t_L} t_R + m_d \overline{d_L} d_R + m_s \overline{s_L} s_R + m_b \overline{b_L} b_R + \text{h.c.}\right)$$

$$-\mathcal{L}_h^h = \frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4$$

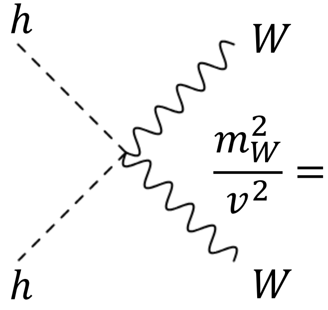


$$\frac{m_h^2}{2v} = \lambda v$$

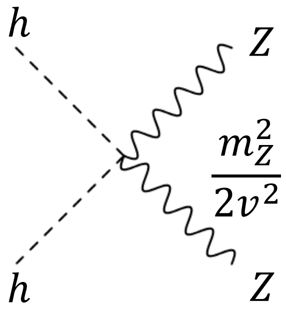


$$\frac{m_h^2}{8v^2} = \frac{\lambda}{4}$$

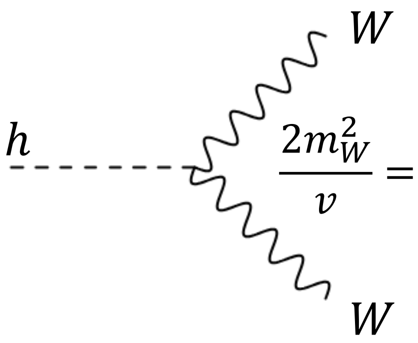
$$-\mathcal{L}_V^h = \left(\frac{2h}{v} + \frac{h^2}{v^2}\right)\left(m_W^2 W_\mu^- W^{\mu +} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)$$



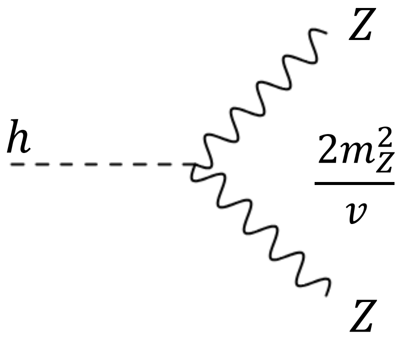
$$\frac{m_W^2}{v^2} = \frac{g^2}{4}$$



$$\frac{m_Z^2}{2v^2} = \frac{g^2 + g'^2}{8}$$



$$\frac{2m_W^2}{v} = \frac{g^2 v}{2}$$

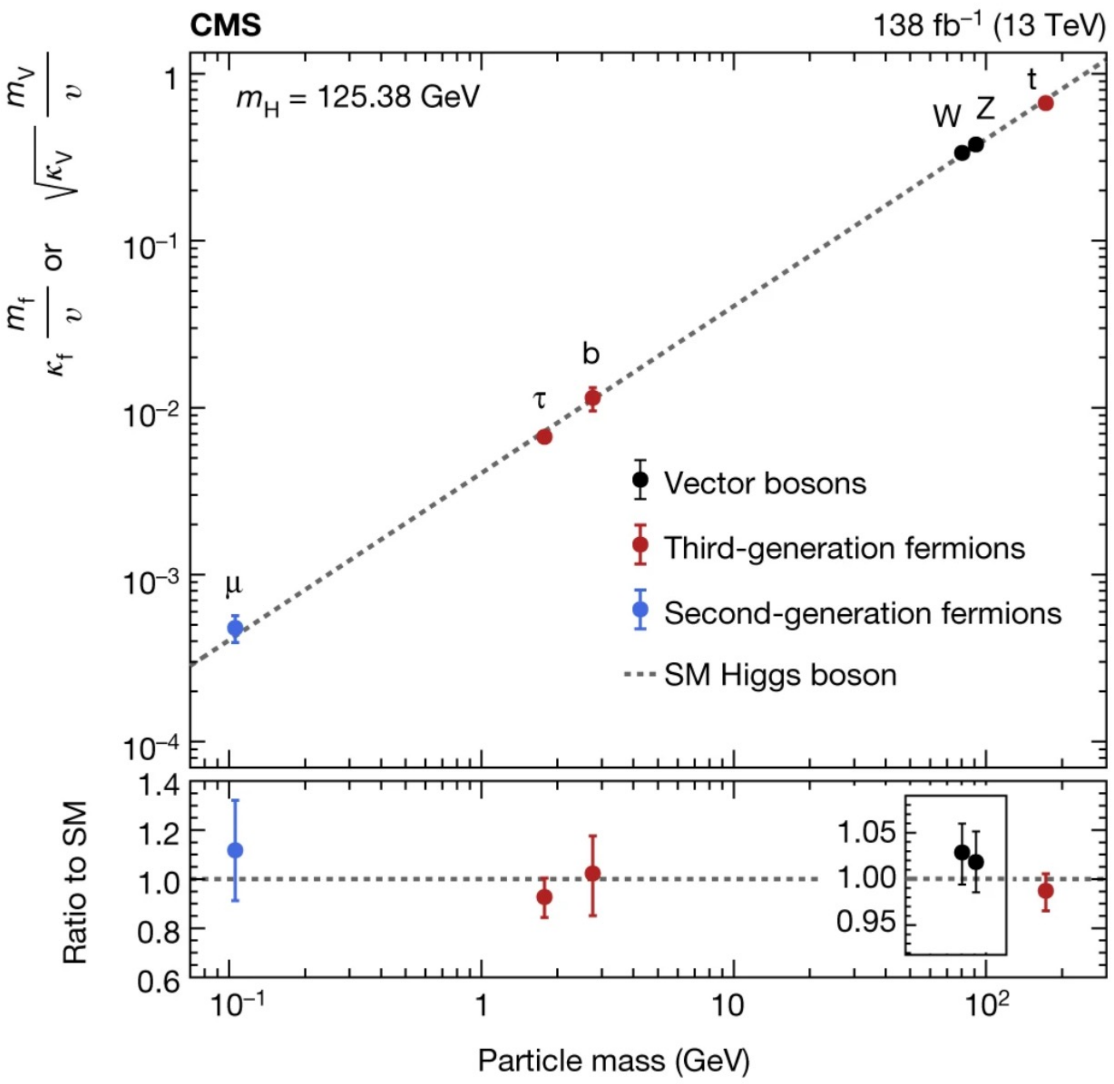


$$\frac{2m_Z^2}{v} = \frac{(g^2 + g'^2)v}{4}$$

All Higgs couplings are proportional to the mass of the particle to which it couples

The Higgs Boson interactions

- Higgs decays are proportional to the mass of the particle to which it couples
- The Higgs decay is dominated by the heaviest particle that can be pair-produced in the decay
- Decays to $t\bar{t}, \gamma\gamma, gg$ are not possible at tree level and can only happen via loops
- Not all Higgs decays are experimentally established
  - at present only $\tau^+\tau^-, b\bar{b}, ZZ^*, WW^*, \gamma\gamma$ are established with rates consistent with the SM predictions



$$BR_{b\bar{b}}: BR_{WW^*}: BR_{gg}: BR_{\tau^+\tau^-}: BR_{ZZ^*}: BR_{c\bar{c}} = 0.58: 0.21: 0.09: 0.06: 0.03: 0.03$$

WW^* and ZZ^* are decays with one boson on-shell and the other off-shell

Neutral current weak interactions

- Z boson couplings to fermions is proportional to

$$g c_W T_3 - g' s_W Y = \frac{\mathrm{g}}{c_W}(T_3 - s_W^2 Q), \qquad c_W = \cos\theta_W\,, \qquad s_W = \sin\theta_W$$

- Using the T_3 and Y assignments of the various fermion fields, we find the following types of Z couplings

$$\mathcal{L}_{Zf\bar{f}} = \frac{g}{c_W}\Big(-\left(\tfrac{1}{2} - s_W^2\right)\overline{e_L^i}\gamma^\mu Z_\mu e_L^i + s_W^2\overline{e_R^i}\gamma^\mu Z_\mu e_R^i + \tfrac{1}{2}\overline{\nu_L^i}\gamma^\mu Z_\mu \nu_L^i + \left(\tfrac{1}{2} - \tfrac{2}{3}s_W^2\right)\overline{u_L^i}\gamma^\mu Z_\mu u_L^i -$$

$$\tfrac{2}{3}s_W^2\overline{u_R^i}\gamma^\mu Z_\mu u_R^i - \left(\tfrac{1}{2} - \tfrac{1}{3}s_W^2\right)\overline{d_L^i}\gamma^\mu Z_\mu d_L^i + \tfrac{1}{3}s_W^2\overline{d_R^i}\gamma^\mu Z_\mu d_R^i\Big)$$

<u>Important features of neutral-current weak interactions (NCWI):</u>

- Z boson couples to neutrinos
- *Parity violation:* Z boson couplings are chiral (LH and RH fields carry different T_3, Z couples to them differently)
- *Diagonality:* the Z boson couple for example to e^+e^-, $\mu^+\mu^-$, $\overline{\nu_{\tau L}}\nu_{\tau L}, \overline{\nu_{\mu L}}\nu_{\mu L}$ but not to $e^\pm\mu^\mp$, $\overline{\nu_{\mu L}}\nu_{\tau L}$ pairs
- *Universality:* the couplings of the Z boson to the different generations are universal

Neutral current weak interactions

Predictions (omitting common and phase space factors)

$$\Gamma(Z \to \nu\bar{\nu}) \propto 1$$

$$\Gamma\left(Z \to l\bar{l}\right) \propto 1 - 4s_W^2 - 8s_W^4$$

$$\Gamma(Z \to u\bar{u}) \propto 3\left(1 - \frac{8}{3}s_W^2 - \frac{32}{9}s_W^4\right)$$

$$\Gamma\left(Z \to d\bar{d}\right) \propto 3\left(1 - \frac{4}{3}s_W^2 - \frac{8}{9}s_W^4\right)$$

$s_W^2 = 0.225$

$$\Gamma_\nu : \Gamma_l : \Gamma_u : \Gamma_d = 1 : 0.51 : 1.74 : 2.24$$

Experimental values

$$BR(Z \to \nu\bar{\nu}) = (6.67 \pm 0.02)\%$$

$$BR\left(Z \to l\bar{l}\right) = (3.366 \pm 0.002)\%$$

$$BR\ (Z \to u\bar{u}) = (11.6 \pm 0.6)\%$$

$$BR\ \left(Z \to d\bar{d}\right) = (15.6 \pm 0.4)\%$$

$$\Gamma_\nu : \Gamma_l : \Gamma_u : \Gamma_d = 1 : 0.505 : 1.74 : 2.34$$

Charged current weak interactions

- Charged $W^{\pm}$ boson couplings to fermions are simple
  - the interaction basis is also mass basis → the W interactions are universal

$$\mathcal{L}_{W,l} = -\frac{g}{\sqrt{2}}\left(\overline{\nu_{eL}}\gamma^{\mu}W_{\mu}^{+}e_{L}^{-} + \overline{\nu_{\mu L}}\gamma^{\mu}W_{\mu}^{+}\mu_{L}^{-} + \overline{\nu_{\tau L}}\gamma^{\mu}W_{\mu}^{+}\tau_{L}^{-} + \text{h.c.}\right)$$

- More complicated in the quark sector
  - no interaction basis that is also a mass basis

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{U_{L}^{i}}\gamma^{\mu}W_{\mu}^{+}D_{L}^{i} + \text{h.c.}$$

$$U_{L}^{i} = \left(V_{uL}^{\dagger}\right)_{ij}u_{L}^{j}, \qquad D_{L}^{i} = \left(V_{dL}^{\dagger}\right)_{ij}d_{L}^{j}$$

⇓

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}\left(V_{uL}\right)_{ki}\gamma^{\mu}W_{\mu}^{+}\left(V_{dL}^{\dagger}\right)_{il}d_{L}^{l} + \text{h.c.} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}V_{kl}\gamma^{\mu}W_{\mu}^{+}d_{L}^{l} + \text{h.c.}$$

↑ CKM matrix (V_{kl})

Charged current weak interactions

$$\mathcal{L}_{W,l} = -\frac{g}{\sqrt{2}}\left(\overline{\nu_{eL}}\gamma^{\mu}W_{\mu}^{+}e_{L}^{-} + \overline{\nu_{\mu L}}\gamma^{\mu}W_{\mu}^{+}\mu_{L}^{-} + \overline{\nu_{\tau L}}\gamma^{\mu}W_{\mu}^{+}\tau_{L}^{-} + \mathrm{h.c.}\right)$$

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}\left(V_{uL}\right)_{ki}\gamma^{\mu}W_{\mu}^{+}\left(V_{dL}^{\dagger}\right)_{il}d_{L}^{l} + \mathrm{h.c.} = -\frac{g}{\sqrt{2}}\overline{u_{L}^{k}}V_{kl}\gamma^{\mu}W_{\mu}^{+}d_{L}^{l} + \mathrm{h.c.}$$

Important features of charged-current weak interactions (CCWI):

- *Parity violation:* only left-handed particles participate in CCWI
- The W couplings to quark mass eigenstates are **NOT universal** the universality of gauge interactions is hidden in the unitarity of the CKM matrix, V
- The W couplings to the quark eigenstates are **NOT diagonal**. This is a consequence of the fact that no pair of up and down type mass eigenstates fits into an $SU(2)$ doublet.

Charged current weak interactions

- Predictions (omitting common factors and phase-space factors)

$$\Gamma(W^+ \to l^+ \nu) \propto 1$$

$$\Gamma\left(W^+ \to u_i \overline{d_j}\right) \propto 3\left|V_{ij}\right|^2, \quad (i = 1, 2;\ j = 1, 2, 3)$$

- The top quark is not included because it is heavier than the W-boson
- Taking into account CKM unitarity relations

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

- We obtain

$$\Gamma(W \to \text{quarks}) \approx 2\Gamma(W \to \text{leptons})$$

- Experimentally:

$$\text{BR}(W \to \text{quarks}) = 67.41(27)\%, \qquad \text{BR}(W \to \text{leptons}) = 32.58(27)\%$$

$$\Gamma(W \to \text{quarks})/\Gamma(W \to \text{leptons}) = 2.07(2)$$

good agreement with SM after including radiative corrections

Charged current weak interactions

- Predictions (omitting common factors and phase-space factors)

$$\Gamma(W^+ \to l^+ \nu) \propto 1$$

$$\Gamma\left(W^+ \to u_i \bar{d}_j\right) \propto 3\left|V_{ij}\right|^2, \quad (i = 1, 2;\ j = 1, 2, 3)$$

- The top quark is not included because it is heavier than the W-boson
- Taking into account CKM unitarity relations

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

- Hidden universality in the quark sector is tested by the prediction

$$\Gamma(W \to uX) = \Gamma(W \to cX) = \frac{1}{2}\Gamma(W \to \text{quarks})$$

- Experimentally:

$$\text{BR}(W \to cX) = 33.3(2.6)\%, \qquad \text{BR}(W \to \text{quarks}) = 67.41(27)\%$$

$$\Gamma(W \to cX)/\Gamma(W \to \text{quarks}) = 0.49(4)$$

The Standard Model fermion interactions

Interaction	Force carrier	Coupling	Fermions
Electromagnetic	γ	eQ	u, d, e
Strong	g	g_s	u, d
NC weak	Z^0	$(g/\cos\theta_W)(T_3 - \sin^2\theta_W Q)$	u, d, e, ν
CC weak for quarks	$W^\pm$	$(g/\sqrt{2})V_{ij}$	$\bar{u}_i d_j$
CC weak for leptons	$W^\pm$	$(g/\sqrt{2})$	$\bar{\nu} e$
Yukawa	h	y_f	u, d, e

Accidental global symmetries

- If we set all Yukawa couplings to zero ($\mathcal{L}_{\text{Yuk}} = 0$), the SM gains a large accidental global symmetry in the fermion sector

$$G_{\text{SM}}^{\text{global}}\left(Y^{u,d,e} = 0\right) = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$$

- Under $U(3)_Q \to (Q_{L1}, Q_{L2}, Q_{L3})$ transform as an $SU(3)_Q$ triplet, all other fields are singlets
- Under $U(3)_U \to (U_{R1}, U_{R2}, U_{R3})$ transform as an $SU(3)_U$ triplet, all other fields are singlets
- Under $U(3)_D \to (D_{R1}, D_{R2}, D_{R3})$ transform as an $SU(3)_D$ triplet, all other fields are singlets
- Under $U(3)_L \to (L_{L1}, L_{L2}, L_{L3})$ transform as an $SU(3)_L$ triplet, all other fields are singlets
- Under $U(3)_E \to (E_{R1}, E_{R2}, E_{R3})$ transform as an $SU(3)_R$ triplet, all other fields are singlets

Accidental global symmetries

- If we sat all Yukawa couplings to zero ($\mathcal{L}_{\text{Yuk}} = 0$), the SM gains a large accidental global symmetry in the fermion sector

$$G_{\text{SM}}^{\text{global}}\left(Y^{u,d,e} = 0\right) = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$$

Breaking due to $Y^{u,d,e} \neq 0$

$$G_{\text{SM}}^{\text{global}} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- Under $U(1)_B \rightarrow$ all quarks (antiquarks) carry the charge $+1/3$ ($-1/3$), while all other fields are neutral
- Baryon number symmetry explains why proton decay has not been observed
- Possible proton decay modes, such as $p \rightarrow e^+\gamma$ are not forbidden by $SU(3)_C \times U(1)_{\text{EM}}$ symmetry but violates baryon number symmetry and therefore does not occur in the SM
- Experimentally: $\tau_{p \rightarrow e^+\gamma} > 6.7 \times 10^{32}$ years

How many parameters do we have in the Standard Model?

- On how many physical parameters does the Standard Model depend?
  - “unphysical” parameters are those that can be set to zero by a basis rotation
- Number of parameters affecting physical measurements (general theorem)

$$N_{\mathrm{phys}} = N_{\mathrm{general}} - N_{\mathrm{broken}}$$

- N_{phys} – number of physical parameters
- N_{tot} – total number of parameters
- N_{broken} – number of broken generators

Example: Zeeman effect

- A hydrogen atom with weak magnetic field
- The magnetic field add one new physical parameter, B

$$V(r) = -\frac{e^2}{r} \quad \Longrightarrow \quad V(r) = -\frac{e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- There are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \rightarrow SO(2)$
- Two broken generators, can be “used” to define the z axis

$$N_{\text{phys}} = N_{\text{general}} - N_{\text{broken}} \quad \Longrightarrow \quad 1 = 3 - 2$$

Back to the flavour sector

- Without the Yukawa interactions, a model with N copies of the same field has a $U(N)$ global symmetry
- Symmetry of the kinetic term

$$\mathcal{L}_{\text{kin}} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, \dots, N$$

- $U(N)$ is a general rotation in $N-$dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators
- Adding terms $\left(\mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_{\text{Yuk}}\right)$ that respect the imposed gauge symmetries, the global symmetry may be broken into a smaller symmetry group
- In breaking the global symmetry there is a freedom to use the broken generators to transform from one interaction basis to another and rotate away unphysical parameters!
- The rule apply separately to real parameters and phases

Counting parameters

$$N_{\text{phys}} = N_{\text{general}} - N_{\text{broken}}$$

- *General $n \times n$ complex matrix:* n^2 real parameters and n^2 phases
- Hermeticity and unitarity reduce the number of parameters required to describe the matrix
- *Hermitian matrix:* $n(n+1)/2$ real parameters and $n(n-1)/2$ phases
- *Unitary matrix $(U(N))$:* $n(n-1)/2$ real parameters and $n(n+1)/2$ phases

- $U(1)$ transformation (which is not a symmetry) can be used to remove a single phase
- $SU(2)$ transformation (which is not a symmetry) can be used to remove $n(n-1)/2$ real parameters together with $n(n+1)/2 - 1$ phases

Counting the Standard Model parameters

$$N_{\text{phys}} = N_{\text{general}} - N_{\text{broken}}$$

- $\mathcal{L}_{\text{kin}}$: three real parameters, the gauge couplings g, g', g_s
- $\mathcal{L}_{\phi}$: two real parameters v, λ
- $\mathcal{L}_{\text{Yuk}}$(lepton sector): three Yukawa couplings for the leptons y_e, y_μ, y_τ
- $\mathcal{L}_{\text{Yuk}}$(quark sector): six Yukawa couplings for the quarks $y_u, y_c, y_t, y_d, y_s, y_b$, three mixing angles + phase
  - two 3×3 complex Yukawa matrices Y^u, Y^d → 36 parameters (18 real parameters and 18 phases) in a general basis
  - the kinetic terms for the quarks have a global symmetry $G_q = U(3)_Q \times U(3)_U \times U(3)_D$ which has 27 generators
  - the Yukawa terms break the symmetry into a baryon number $H_q = U(1)_B$, which has a single generator → $N_{\text{broken}} = 26$

$$N_{\text{phys}} = 36 - 26 = 10 \Longrightarrow \qquad N_{\text{phys}}^{(r)} = 18 - 9 = 9 \qquad N_{\text{phys}}^{(i)} = 18 - 17 = 1$$

- SM has **18 parameters:** 3 gauge couplings, 2 related to the Higgs potential, 3 charged lepton masses, 6 quark masses, and 4 CKM parameters

Summary of Lecture 5

Main learning outcomes

- Which interactions do we have between the Standard Model fields
- What are the global accidental symmetries of the Standard Model
- How to do parameter counting in the Standard Model: physical parameters and broken generators